文章编号：1000-4750(2013)02-0038-06

一维正方准晶椭圆孔口反平面问题的半逆解法

刘官厅，冯中华
(内蒙古师范大学数学科学学院，呼和浩特 010022)

摘　要： 选取新的位移势函数，利用半逆解法及待定系数法，研究一维正方准晶平行于准周期方向的椭圆孔口问题，给出了应力场的显式解析解。在极限状态下，椭圆孔口问题可退化为 Griffith 裂纹问题，得到了相应裂纹问题的应力场和应力强度因子的显式解析解。

关键词： 一维正方准晶；椭圆孔口；应力强度因子；半逆解法；显式解析解

中图分类号： O346.1　**文献标志码：** A　doi: 10.6052/j.issn.1000-4750.2011.07.0455

HALF-INVERSE METHOD FOR THE ANTI-PLANE PROBLEM OF ONE-DIMENSIONAL ORTHORHOMBIC QUASI-CRYSTALS WITH ELLIPTICAL HOLE

LIU Guan-ting , FENG Zhong-hua
(College of Mathematical Science, Inner Mongolia Normal University, Hohhot 010022, China)

Abstract: Choosing a new displacement potential function and using a half-inverse method and an undetermined coefficient method, the problem about one-dimensional orthorhombic quasi-crystals with an elliptical hole in parallel to the quasi-periodic direction is investigated, the explicit analytical solutions in stress field is given. Under the limiting conditions, the problem of an elliptical hole degenerates into the problem of a Griffith crack. The explicit analytical solutions in stress field and the stress field intensity factor are obtained corresponding to the crack problem.

Key words: one-dimensional orthorhombic quasi-crystals; elliptical hole; stress field intensity factor; half-inverse method; explicit analytical solutions

准晶是 1984 年被发现的一种新固体结构和新材料，自准晶被发现以来，关于准晶弹性和缺陷方面的研究已取得很多成果[1−4]，由于一维六方准晶的弹性常数比较少，故结构相对简单，所以成为了许多研究者的首选对象，取得了一些成果[5−8]。一维正方准晶的弹性常数比一维六方准晶的弹性常数多，故一维正方准晶的弹性问题较一维六方准晶更加复杂。半逆解法是求解断裂力学问题的重要方法之一，而选取位移势函数是半逆解法的关键，也是半逆解法的难点，需针对不同的缺陷问题选择不同的位移势函数，杨维阳等人运用半逆解法求解裂纹问题，取得很多成果[9−11]，而运用半逆解法求解椭圆孔口问题并不常见。文献[12]通过构造广义保角映射研究了一维正方准晶椭圆孔口平面弹性问题，只是得到该问题应力场的复变表示，通过该复变表示不能直接看出应力场与弹性常数的关系。本文选取新的位移势函数，利用半逆解法和待定系数法，研究一维正方准晶平行于准周期方向的椭圆孔口问题，与文献[12]相比，本文给出了依赖于弹性常数的应力场的显式解析解，能直接看出应力场与弹性常数的关系。且在极限状态下，椭圆孔口问题可退化为 Griffith 裂纹问题，得到相应裂纹问题的应力场和应力强度因子的显式解析解。

收稿日期：2011-07-17；修改日期：2012-02-22
基金项目：国家自然科学基金项目(11262017)
通讯作者：刘官厅(1966－)，男，内蒙古包头人，教授，博士，主要从事断裂力学研究(E-mail: guantingliu@imnu.edu.cn).
作者简介：冯中华(1986－)，女，内蒙古赤峰人，硕士生，从事连续介质力学中的数学问题研究(E-mail: fengzhonghua2005@163.com).

1　预备知识

由文献[5]知，取一维正方准晶的准周期方向为坐标轴 x_3，垂直于准周期方向的平面为坐标平面 x_1-x_2，建立直角坐标系，当一维正方准晶的缺陷沿准周期方向穿透时，材料的几何性质将不随准周期方向改变，则有：

$$\frac{\partial}{\partial x_3}=0 \tag{1}$$

将式(1)分别代入到广义胡克定律、平衡方程和几何方程，此时可分为两个互相分离的问题。其中一个问题与经典问题类似，这里不作讨论，另一个问题为：

$$\sigma_{23}=\sigma_{32}=2C_{44}\varepsilon_{23}+R_5 w_{32} \tag{2}$$

$$\sigma_{13}=\sigma_{31}=2C_{55}\varepsilon_{31}+R_6 w_{31} \tag{3}$$

$$H_{31}=2R_6\varepsilon_{31}+K_1 w_{31} \tag{4}$$

$$H_{32}=2R_5\varepsilon_{23}+K_2 w_{32} \tag{5}$$

$$\varepsilon_{3j}=\varepsilon_{j3}=\frac{1}{2}\partial_j u_3,\ w_{3j}=\partial_j w,\ j=1,2 \tag{6}$$

$$\partial_1\sigma_{31}+\partial_2\sigma_{32}=0,\ \partial_1 H_{31}+\partial_2 H_{32}=0 \tag{7}$$

将式(6)代入式(2)~式(5)，然后将所得结果代入式(7)，得：

$$(C_{55}\partial_1^2+C_{44}\partial_2^2)u_3+(R_6\partial_1^2+R_5\partial_2^2)w=0 \tag{8}$$

$$(R_6\partial_1^2+R_5\partial_2^2)u_3+(K_1\partial_1^2+K_2\partial_2^2)w=0 \tag{9}$$

引入位移势函数 $F(x_1,x_2)$，使得：

$$u_3=(R_6\partial_1^2+R_5\partial_2^2)F,\ w=-(C_{55}\partial_1^2+C_{44}\partial_2^2)F \tag{10}$$

此时式(8)自动满足，把式(10)代入式(9)，得：

$$[k_1\partial_1^4+k_3\partial_1^2\partial_2^2+k_5\partial_2^4]F=0 \tag{11}$$

其中：$k_1=R_6^2-K_1C_{55}$，$k_5=R_5^2-K_2C_{44}$，$k_3=2R_5R_6-K_1C_{44}-K_2C_{55}$。

2　一维正方准晶平行于准周期方向的椭圆孔口的应力分析

在一维正方准晶中，如图 1 所示，沿准周期 x_3 方向有一穿透性的椭圆孔口，设椭圆孔口边界的方程为 $x_1=a\cos\theta,\ \ x_2=b\sin\theta$。考虑在无穷远处沿准周期 x_3 方向作用均匀外载荷 P，椭圆孔口上不受力的静平衡问题。

线弹性理论分析的结果表明，如果准晶的无穷远处不受力，而仅在椭圆孔口的表面受 $\sigma_{23}=-P$ 作用的静平衡问题，该问题与前一个问题所得结果除去一个常数项外是等价的。现考虑前一问题，其边界条件为：

$$x_1\to\infty,\ x_2\to\infty,\ \sigma_{23}=P,\ \sigma_{31}=0,\ H_{32}=H_{31}=0,$$

$$x_1=a\cos\theta,\ \ x_2=b\sin\theta,\ \sigma_{23}=0,\ \ H_{32}=0。$$

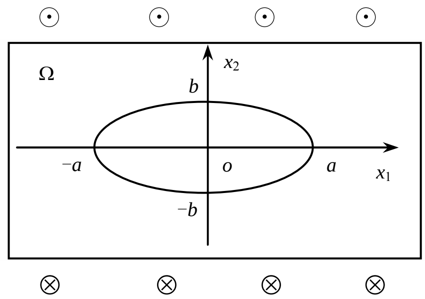

图 1　一维正方准晶中沿准周期方向的椭圆孔口

Fig.1　Elliptical hole in one-dimensional orthorhombic quasi-crystals along the quasi-periodic direction

假设位移势函数 $F=F(x_1+\mu x_2)$，将其代入式(11)，得到特征方程为：

$$k_5\mu^4+k_3\mu^2+k_1=0$$

由文献[13]知，

当 $\Delta>0$ 时，此方程的解为 $\mu_1=\mathrm{i}\beta_1,\ \ \mu_2=\mathrm{i}\beta_2,\ \ \mu_3=\bar{\mu}_1,\ \ \mu_4=\bar{\mu}_2$，其中 $\beta_2>\beta_1>0$，且为实数。

当 $\Delta<0$ 时，此方程的解为 $\mu_1=\alpha+\mathrm{i}\beta,\ \ \mu_2=-\alpha+\mathrm{i}\beta,\ \ \mu_3=\bar{\mu}_1,\ \ \mu_4=\bar{\mu}_2$，其中 $\beta>\alpha>0$，且为实数。

此时式(11)可化为 $\nabla_1^2\nabla_2^2F=0$，其对应的实值解析解为：

$$F=\sum_{j=1}^{2}[a_j\mathrm{Re}(\bar{\bar{\bar{F}}}_j)+b_j\mathrm{Im}(\bar{\bar{\bar{F}}}_j)] \tag{12}$$

其中 $\bar{\bar{\bar{F}}}_j=\bar{\bar{\bar{F}}}_j(z_j),\ \ \bar{\bar{F}}_j=\bar{\bar{F}}_j(z_j),\ \ \bar{F}_j=\bar{F}_j(z_j),\ \ \frac{\mathrm{d}\bar{\bar{\bar{F}}}_j}{\mathrm{d}z_j}=\bar{\bar{F}}_j,\ \ \frac{\mathrm{d}\bar{\bar{F}}_j}{\mathrm{d}z_j}=\bar{F}_j,\ \ \frac{\mathrm{d}\bar{F}_j}{\mathrm{d}z_j}=F_j,\ \ z_j=x_1+\mu_j x_2$，$a_j$、$b_j$ 为待定的系数，$j=1,2$。

把式(12)代入式(10)，将所得结果代入式(6)，再将所得结果代入式(2)和式(5)，得：

$$\sigma_{23}=C_{44}R_6\sum_{j=1}^{2}[a_j\mathrm{Re}(\mu_jF_j)+b_j\mathrm{Im}(\mu_jF_j)]-R_5C_{55}\sum_{j=1}^{2}[a_j\mathrm{Re}(\mu_jF_j)+b_j\mathrm{Im}(\mu_jF_j)] \tag{13}$$

$$H_{32}=R_5R_6\sum_{j=1}^{2}[a_j\mathrm{Re}(\mu_jF_j)+b_j\mathrm{Im}(\mu_jF_j)]+R_5^2\sum_{j=1}^{2}[a_j\mathrm{Re}(\mu_j^3F_j)+b_j\mathrm{Im}(\mu_j^3F_j)]-K_2C_{55}\sum_{j=1}^{2}[a_j\mathrm{Re}(\mu_jF_j)+b_j\mathrm{Im}(\mu_jF_j)]-$$

$$K_2C_{44}\sum_{j=1}^{2}[a_j\mathrm{Re}(\mu_j^3F_j)+b_j\mathrm{Im}(\mu_j^3F_j)] \tag{14}$$

选取新的位移势函数：

$$F_j=\frac{P[z_j^2+z_j\sqrt{z_j^2-(a^2+\mu_j^2b^2)}-\mu_j^2b^2-\mathrm{i}ab\mu_j]}{z_j^2+z_j\sqrt{z_j^2-(a^2+\mu_j^2b^2)}-a^2+\mathrm{i}ab\mu_j},\quad j=1,2 \tag{15}$$

当 $x_1\to\infty$, $x_2\to\infty$, $F_j=P$, $j=1,2$；

$x_1=a\cos\theta$, $x_2=b\sin\theta$, $F_1=F_2=-\mathrm{i}P\cot\theta$。

1) 当 $\Delta>0$ 时，把式(13)和式(14)代入边界条件并化简，得：

$$\begin{cases}(\beta_1b_1+\beta_2b_2)(C_{44}R_6-R_5C_{55})=1\\ \beta_1[(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})]b_1+\\ \quad\beta_2[(R_5R_6-K_2C_{55})-\beta_2^2(R_5^2-K_2C_{44})]b_2=0\\ \beta_1a_1+\beta_2a_2=0\\ \beta_1[(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})]a_1+\\ \quad\beta_2[(R_5R_6-K_2C_{55})-\beta_2^2(R_5^2-K_2C_{44})]a_2=0\end{cases}$$

解这个方程组，得：

$a_1=a_2=0$,

$$b_1=\frac{(R_5^2-K_2C_{44})\beta_2^2-(R_5R_6-K_2C_{55})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)\beta_1},$$

$$b_2=\frac{(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)\beta_2}。$$

把 a_1、a_2、b_1、b_2 所得的值代入式(13)和式(14)，得：

$$\begin{aligned}\sigma_{23}={}&C_{44}R_6P\cdot\left[\frac{(R_5^2-K_2C_{44})\beta_2^2-(R_5R_6-K_2C_{55})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_1+\frac{(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_2\right]-\\&R_5C_{55}P\cdot\left[\frac{(R_5^2-K_2C_{44})\beta_2^2-(R_5R_6-K_2C_{55})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_1+\frac{(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_2\right]\end{aligned} \tag{16}$$

$$\begin{aligned}H_{32}={}&R_5R_6P\cdot\left[\frac{(R_5^2-K_2C_{44})\beta_2^2-(R_5R_6-K_2C_{55})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_1+\frac{(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_2\right]+\\&R_5^2P\cdot\left[-\frac{(R_5^2-K_2C_{44})\beta_1^2\beta_2^2-(R_5R_6-K_2C_{55})\beta_1^2}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_1-\frac{\beta_2^2(R_5R_6-K_2C_{55})-\beta_1^2\beta_2^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_2\right]-\\&R_5C_{55}P\cdot\left[\frac{(R_5^2-K_2C_{44})\beta_2^2-(R_5R_6-K_2C_{55})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_1+\frac{(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_2\right]-\\&R_5C_{44}P\cdot\left[-\frac{(R_5^2-K_2C_{44})\beta_1^2\beta_2^2-(R_5R_6-K_2C_{55})\beta_1^2}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_1-\frac{\beta_2^2(R_5R_6-K_2C_{55})-\beta_1^2\beta_2^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_2\right]\end{aligned} \tag{17}$$

其中：

$$g_1=\mathrm{Re}\frac{z_1^2+z_1\sqrt{z_1^2-a^2+\beta_1^2b^2}+b^2\beta_1^2+ab\beta_1}{z_1^2+z_1\sqrt{z_1^2-a^2+\beta_1^2b^2}-a^2-ab\beta_1},$$

$$g_2=\mathrm{Re}\frac{z_2^2+z_2\sqrt{z_2^2-a^2+\beta_2^2b^2}+b^2\beta_2^2+ab\beta_2}{z_2^2+z_2\sqrt{z_2^2-a^2+\beta_2^2b^2}-a^2-ab\beta_2}$$

当 $b=0$ 时，一维正方准晶椭圆孔口可退化为一维正方准晶 Griffith 裂纹，则式(16)和式(17)可转化为：

$$\begin{aligned}\sigma_{23}={}&C_{44}R_6P\cdot\left[\frac{(R_5^2-K_2C_{44})\beta_2^2-(R_5R_6-K_2C_{55})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_3+\frac{(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_4\right]-\\&R_5C_{55}P\cdot\left[\frac{(R_5^2-K_2C_{44})\beta_2^2-(R_5R_6-K_2C_{55})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_3+\frac{(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_4\right]\end{aligned} \tag{18}$$

$$\begin{aligned}H_{32}={}&R_5R_6P\cdot\left[\frac{(R_5^2-K_2C_{44})\beta_2^2-(R_5R_6-K_2C_{55})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_3+\frac{(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_4\right]+\\&R_5^2P\cdot\left[-\frac{(R_5^2-K_2C_{44})\beta_1^2\beta_2^2-\beta_1^2(R_5R_6-K_2C_{55})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_3-\right.\end{aligned}$$

$$\frac{(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_4\Bigg]-$$

$$R_5C_{55}P\cdot$$

$$\Bigg[\frac{(R_5^2-K_2C_{44})\beta_2^2-(R_5R_6-K_2C_{55})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_3-$$

$$\frac{(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_4\Bigg]-$$

$$R_5C_{44}P\cdot$$

$$\Bigg[-\frac{(R_5^2-K_2C_{44})\beta_1^2\beta_2^2-\beta_1^2(R_5R_6-K_2C_{55})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_3-$$

$$\frac{(R_5R_6-K_2C_{55})-\beta_1^2(R_5^2-K_2C_{44})}{(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})(\beta_2^2-\beta_1^2)}g_4\Bigg]$$

其中：$g_3=\mathrm{Re}\dfrac{z_1}{(z_1^2-a^2)^{\frac{1}{2}}}$，$g_4=\mathrm{Re}\dfrac{z_2}{(z_2^2-a^2)^{\frac{1}{2}}}$。

若定义III型应力强度因子为：

$$K_{\mathrm{III}}^{\parallel}=\lim_{x_1\to a}\sqrt{2\pi}(x_1-a)^{\frac{1}{2}}\sigma_{23}$$

则由式(18)，得：

$$K_{\mathrm{III}}^{\parallel}=P\sqrt{\pi a}$$

得到声子场应力强度因子与文献[2]和文献[12]得到的情况一致。也进一步说明一维正方准晶的声子场应力强度因子与经典断裂力学给出的应力强度因子一致。

2) 当$\Delta<0$时，把式(13)和式(14)代入边界条件并化简，得：

$$\begin{cases}[(a_1-a_2)\alpha+(b_1+b_2)\beta](C_{44}R_6-R_5C_{55})=1\\ (a_1-a_2)[(R_5R_6-K_2C_{55})\alpha+(R_5^2-K_2C_{44})\cdot\\ \quad(\alpha^3-3\alpha\beta^2)]=-(b_1+b_2)[(R_5R_6-K_2C_{55})\beta+\\ \quad(R_5^2-K_2C_{44})(3\alpha^2\beta-\beta^3)]\\ (a_1+a_2)\beta+(b_2-b_1)\alpha=0\\ (a_1+a_2)[\beta(R_5R_6-K_2C_{55})+(3\alpha^2\beta-\beta^3)(R_5^2-\\ \quad K_2C_{44})]=-(b_2-b_1)[\alpha(R_5R_6-K_2C_{55})+(\alpha^3-\\ \quad 3\alpha\beta^2)(R_5^2-K_2C_{44})]\end{cases}$$

解这个方程组，得：

$$a_1=-a_2=\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)}{4\alpha(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}$$

$$b_1=b_2=-\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(\alpha^2-3\beta^2)}{4\beta(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}$$

把a_1、a_2、b_1、b_2所得的值代入到式(13)和式(14)，得：

$$\sigma_{23}=C_{44}R_6P\cdot$$

$$\Bigg[\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_5-$$

$$\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(\alpha^2-3\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_6\Bigg]-$$

$$R_5C_{55}P\cdot$$

$$\Bigg[\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_5-$$

$$\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(\alpha^2-3\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_6\Bigg]\qquad(19)$$

$$H_{32}=$$

$$R_5R_6P\cdot\Bigg[\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_5-\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(\alpha^2-3\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_6\Bigg]+$$

$$R_5^2P\Bigg\{\frac{(\alpha^3-3\alpha\beta^2)[(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)]}{4\alpha(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_5-$$

$$\frac{(3\alpha^2\beta-\beta^3)[(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(\alpha^2-3\beta^2)]}{4\beta(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_6\Bigg\}-$$

$$K_2C_{55}P\Bigg[\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_5-\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(\alpha^2-3\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_6\Bigg]-$$

$$K_2C_{44}P\frac{(\alpha^3-3\alpha\beta^2)[(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)]}{4\alpha(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_5-$$

$$\frac{(3\alpha^2\beta-\beta^3)[(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})\cdot(\alpha^2-3\beta^2)]}{4\beta(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_6\Bigg\}\qquad(20)$$

其中：

$$g_5 = \mathrm{Re}\left[\frac{z_1^2+z_1\sqrt{z_1^2-a^2-b^2m_1^2}-b^2m_1^2-\mathrm{i}abm_1}{z_1^2+z_1\sqrt{z_1^2-a^2-b^2m_1^2}-a^2+\mathrm{i}abm_1}+\frac{z^2+z_2\sqrt{z_2^2-a^2-b^2m_2^2}-b^2m_2^2-\mathrm{i}abm_2}{z_2^2+z_2\sqrt{z_2^2-a^2-b^2m_2^2}-a^2+\mathrm{i}abm_2}\right],$$

$$g_6 = \mathrm{Im}\left[\frac{z_1^2+z_1\sqrt{z_1^2-a^2-b^2m_1^2}-b^2m_1^2-\mathrm{i}abm_1}{z_1^2+z_1\sqrt{z_1^2-a^2-b^2m_1^2}-a^2+\mathrm{i}abm_1}+\frac{z_2^2+z_2\sqrt{z_2^2-a^2-b^2m_2^2}-b^2m_2^2-\mathrm{i}abm_2}{z_2^2+z_2\sqrt{z_2^2-a^2-b^2m_2^2}-a^2+\mathrm{i}abm_2}\right],$$

$m_1=\alpha+\mathrm{i}\beta,\ \ m_2=-\alpha+\mathrm{i}\beta,$

$m_1^2=\alpha^2-\beta^2+2\mathrm{i}\alpha\beta,\ \ m_2^2=\alpha^2-\beta^2-2\mathrm{i}\alpha\beta$

当$b=0$时，一维正方准晶椭圆孔口可退化为一维正方准晶 Griffith 裂纹，则式(19)和式(20)可转化为：

$$\sigma_{23}=C_{44}R_6P\cdot\left[\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_7-\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(\alpha^2-3\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_8\right]-R_5C_{55}P\cdot\left[\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_7-\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(\alpha^2-3\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_8\right] \quad (21)$$

$$H_{32}=R_5R_6P\cdot\left[\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_7-\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(\alpha^2-3\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_8\right]+$$
$$R_5^2P\left\{\frac{(\alpha^3-3\alpha\beta^2)[(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)]}{4\alpha(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_7-\frac{(3\alpha^2\beta-\beta^3)[(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(\alpha^2-3\beta^2)]}{(4\beta(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_8\right\}-$$
$$K_2C_{55}P\left[\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_7-\frac{(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(\alpha^2-3\beta^2)}{4(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_8\right]-$$
$$K_2C_{44}P\left\{\frac{(\alpha^3-3\alpha\beta^2)[(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(3\alpha^2-\beta^2)]}{4\alpha(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_7-\frac{(3\alpha^2\beta-\beta^3)[(R_5R_6-K_2C_{55})+(R_5^2-K_2C_{44})(\alpha^2-3\beta^2)]}{4\beta(\alpha^2+\beta^2)(C_{44}R_6-R_5C_{55})(R_5^2-K_2C_{44})}g_8\right\}$$

其中：

$$g_7=\mathrm{Re}\left[\frac{z_1}{({z_1}^2-a^2)^{\frac{1}{2}}}+\frac{z_2}{({z_2}^2-a^2)^{\frac{1}{2}}}\right],$$

$$g_8=\mathrm{Im}\left[\frac{z_1}{({z_1}^2-a^2)^{\frac{1}{2}}}+\frac{z_2}{({z_2}^2-a^2)^{\frac{1}{2}}}\right]$$

若定义III型应力强度因子为：

$$K_{\mathrm{III}}^{\parallel}=\lim_{x_1\to a}\sqrt{2\pi}(x_1-a)^{\frac{1}{2}}\sigma_{23}$$

则由式(21)，得：

$$K_{\mathrm{III}}^{\parallel}=P\sqrt{\pi a}$$

得到声子场应力强度因子与文献[2]和文献[12]得到的情况一致。也进一步说明一维正方准晶的声子场应力强度因子与经典断裂力学给出的应力强度因子一致。

3 结论

本文选取新的位移势函数，利用半逆解法和待定系数法，研究了一维正方准晶椭圆孔口问题的半逆解法，得到应力场的显式解析解。在极限状态下，椭圆孔口问题可退化为 Griffith 裂纹问题，得到了相应裂纹问题的应力场和应力强度因子的显式解析解，其中声子场应力强度因子与文献[2]和文献[12]完全一致，文献[12]仅得到依赖于特征根的隐式应力场解析解，而本文得到依赖于弹性常数的显式应力场解析解，比文献[12]研究更进了一步，因此，本文推广了文献[12]的结果。

参考文献：

[1] Fan Tianyou. A study on the specific heat of a one-dimensional quasicrystal [J]. Journal of Physics Condensed Matter, 1991, 11(45): 513－517.

[2] 范天佑. 准晶数学弹性理论及应用[M]. 北京: 北京理工大学出版社, 1999: 1－24.
Fan Tianyou. Theory and application of the mathematic elasticity about quasicrystal [M]. Beijing: Institute of Technology Press, 1999: 1－24. (in Chinese)

[3] Li Xianfang, Fan Tianyou. A straight dislocation in one dimensional hexagonal quasicrystals [J]. Physica Status Solidi B, 1999, 212(1): 19－26.

[4] Liu Guanting, Guo Ruiping, Fan Tianyou. On the interaction between dialocations and cracks in one-dimensional hexagonal quasicrystals [J]. Chinese Physics, 2003, 2(10): 1149－1155.

[5] 刘官厅. 准晶弹性的复变方法与非线性发展方程的显示解[M]. 呼和浩特: 内蒙古人民出版社, 2005: 21－30.
Liu Guanting. The complex variable function method of quasicrystals elasticity and analytic solutions of nonlinear equations [M]. Hohhot: Inner Mongolia People Press, 2005: 21－30. (in Chinese)

[6] 郭俊宏, 刘官厅. 一维六方准晶中具有不对称裂纹的圆形孔口问题的解析解[J]. 应用数学学报, 2007, 30(6): 1066－1075.
Guo Junhong, Liu Guanting. Analytic solutions of the one-dimensional hexagonal quasicrystals about problem of a circular hole with asymmetry cracks [J]. Acta Mathematicae Applicatae Sinica, 2007, 30(6): 1066－1075. (in Chinese)

[7] 郭俊宏, 刘官厅. 一维六方准晶中带双裂纹的椭圆孔口问题的解析解[J]. 应用数学和力学, 2008, 29(4): 439－446.
Guo Junhong, Liu Guanting. Analytic solutions of problem about an elliptic hole with two straight cracks in one-dimensional hexagonal quasicrystals [J]. Applied Mathematics and Mechanics, 2008, 29(4): 439－446. (in Chinese)

[8] 杨丽星, 刘官厅. 一维六方准晶中带三条不对称裂纹的圆形孔口问题的解析解[J]. 数学的实践与认识, 2010, 40(2): 148－156.
Yang Lixing, Liu Guanting. Analytic solutions of problem about a circular hole with three unequal straight edge-cracks in one-dimensional hexagonal quasicrytals [J]. Mathematics in Practices and Theory, 2010, 40(2): 148－156. (in Chinese)

[9] 谢秀峰, 李俊林, 杨维阳. 正交异性复合材料I型裂纹尖端应力场研究[J]. 科学技术与工程, 2008, 8(7): 1780－1794.
Xie Xiufeng, Li Junlin, Yang Weiyang. Mode I crack stress field near crack tip of orthotropic composite material [J]. Science Technology and Engineering, 2008, 8(7): 1780－1794. (in Chinese)

[10] 李俊林, 张少琴, 杨维阳. 正交异性复合材料板界面裂纹尖端应力强度因子[J]. 中北大学学报, 2006, 27(5): 380－383.
Li Junlin, Zhang Shaoqin, Yang Weiyang. Stress intensity factors near interface crack tip of two dissimilar orthotropic composite material [J]. Journal of North University of China, 2006, 27(5): 380－383. (in Chinese)

[11] 张雪霞, 崔小朝, 杨维阳, 等. 正交异性双材料的II型界面裂纹尖端场[J]. 应用数学和力学, 2009, 30(12): 1399－1414.
Zhang Xuexia, Cui Xiaochao, Yang Weiyang, et al. Crack-tip field on mode II interface crack of double dissimilar orthotropic composite materials [J]. Applied Mathematics and Mechanics, 2009, 30(12): 1399－1414. (in Chinese)

[12] 于静, 刘官厅. 一维正方准晶椭圆孔口平面弹性问题的解析解[J]. 固体力学学报, 2010, 31(4): 411－416.
Yu Jing, Liu Guanting. Analytic solutions of plane elasticity of one-dimensional orthorhombic quasicrystals with elliptical hole [J]. Chinese Journal of Solid Mechanics, 2010, 31(4): 411－416. (in Chinese)

[13] 杨维阳, 李俊林, 张雪霞. 复合材料断裂复变方法[M]. 北京: 科学出版社, 2005: 26－27.
Yang Weiyang, Li Junlin, Zhang Xuexia. Method of a complex variable for fracture in composite materials [M]. Beijing: Science Press, 2005: 26－27. (in Chinese)